Estimating fund manager fees using option pricing model

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Abstract: This paper analyses the private equity fund compensation. We build a model to estimate the expected revenue of fund managers as a function of their investor contracts. We tried to evaluate the present value of the carried interest, which is one of the most common profit sharing arrangements observed in private equity funds. We applied the Monte Carlo simulation model and we introduced the non-marketability discount of the carried interest in order to calculate its fair value. The purpose of this paper is to determine the sensitivity of the expected carried interest fair value to the variation of its optional parameters.

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Introduction

Typically, investment funds are organized as limited partnerships, professionals in private equity funds are considered as general partners (GPs), while institutional investors and other wealthy investors who provide the remaining capital are considered as limited partners (LPs).

The GPs act as professional intermediaries by reducing uncertainty, asymmetric information and agency costs incurred when institutions invest directly in new ventures. To align the interests of LPs and GPs, in a typical structure of funds, the compensation of GPs is highly dependent on the commercial success of the fund and its performance. The GP usually receives about 20% of fund's net profits (Litvak [2004]; Sahlman [1990]; Johan and Najar [2011]). In addition, before being allowed to receive any distribution of earnings, GPs are often contractually obliged to return to their LPs the total capital invested, plus a preferred return required by investors, namely the "hurdle rate"(Gompers and Lerner [1999]; Wahrenburg and Schmidt [2003]; Metrick and Yasuda [2010]). The GP usually receives an annual management fee of about 2-2.5% of total committed capital of the fund. These fees are collected primarily to cover the costs of management and investment activities conducted by the manager of the fund. These income received by GPs can decrease until the end of the life’s fund to reflect the gradual reduction of the activities of GPs.

Therefore, the compensation of GPs consists of two parts: a fixed and a variable part. The fixed part representing the “management fees” can be considered as compensation for services in mutual funds and hudge-fund. Moreover, the variable part cannot be equated with variable annuities issued by hedge funds. This part known as Interest Performance Fee or “carried interest” represents a percentage of the profits earned by the fund.

This part of the compensation has been the subject of several reform studies on their tax treatment. The fact of imposing the carried interest at the grant date requires the determination of its fair value, but this is not an easy task. Previous work mainly in tax literature proposes to approximate the carried interest to a call option that gives to its holder (GP) the right to receive a portion of the fund’s profit. Some researches propose to apply the most common models of option pricing such as Black-Scholes model.

Otherwise, the carried interest can be considered as a call option but this option is quite specific as it is granted under certain conditions. Thus we will focus on the particularity of these options that has not been considered in previous work. The carried interest are neither directly transferable to someone else nor freely tradable in the open market. Under such circumstance, it can be argued based on sound financial and economic theory that non marketability and non transferability discount can be appropriately applied to the carried interest. We propose in this work to apply the appropriate models of option pricing to determine the fair value of the carried interest while taking into account the criterion of non-transferability.

Initially, we present a literature review about defining and evaluating fund manager compensation, than we determine in the second section of the paper, the various optional characteristics of the carried interest that can justify the use of option pricing models. In the third part, we introduce the non-marketability criteria of the carried interest that will reduce its fair value and finally we perform an empirical evaluation of the carry. In this part of the paper, we analyze the sensitivity of carried interest fair value according to the different fund characteristics and the distribution terms.

1. Previous attempts to evaluate GPs compensation

Several research studies have focused on partners incomes of investment funds. Gompers and Lerner (1999) were only interested in the fund venture capital. They explore the sectoral variation and temporal conditions of the funds. Litvak (2004) conduct a study larger than that of Gompers and Lerner (1999) by introducing additional conditions for partners. Otherwise, none of this work examines buyout funds. Conner (2005) uses a simulation to estimate the different pricing terms, but it uses outlook ex-post (which require some forecasts on income funds) rather than ex-ante perspective based on an equilibrium relationship.

Phalippou and Gottschalg (2007) precise that it is difficult to assess the level of fund manager compensation, composed with a fixed part (management fees), a variable part (carried interest), plus additional compensation (such as transaction fees). Phalippou (2007) considers that there is certain opacity in compensation, and it is sometimes difficult to define its contours. For every $ 100 of committed capital, the discounted amount of compensation is $ 24.18 dollars for VC funds and 17.29 dollars for BO funds. On an annual basis, Gottschalg, Kreuter and Phalippou (2007) result in compensation per employee (professional) of around 1 million euro.

Several studies have focused on determining the relationship between wages in the investment fund and the fund's historical performance. Chevalier and Ellison (1997), Sirri and Tufano (1998), Barclay, Pearson and Weisbach (1998), and Sensoy (2009) have conducted regressions flows of mutual funds based on the historical performance of funds and they found a positive relationship between these two nonlinear variables.

Moreover, Metrick and Yasuda (2010) regressed the amount of remuneration per dollar, per associate per employee on the characteristics of the fund in terms of experience and past performance of GP. In these regressions, the performance is never significant, implying that GPs performing their pay increase by increasing the size of funds they manage, rather than wage levels. The GP's professional experience positively influences pay per partner and per employee, but not compensation per dollar invested. According to the results of Metrick and Yasuda (2010), it does not appear that the fixed and variable components of remuneration behave differently based on past performance of funds. This confirms the remark made by the Treasury Committee of the House of Commons and therefore suggests that the perception of a fixed commission invariant does not contribute to aligning the interests of LPs and GPs.

In the literature, the "carried interest" can be likened to a call option on the profits of a limited partnership. Thus we can apply the option pricing methods, such as Black-Scholes, to assess the value of "carried interest". In this case, they use a risk-neutral evaluation of a call option that represents the carry. On the other hand, they perform a simulation to obtain the exit dates and yields of each investment fund.
Theoretically, the Black-Scholes formula used to evaluate the premium of a call option, calculate the difference between the exercise price paid at maturity, and the current value of assets that it will reach maturity (taking into account the volatility of the underlying asset).

Moreover, there are some difficulties in applying option pricing techniques, particularly Black-Scholes formula because it suppose that we know exactly the value of each parameter (volatility, maturity…) which is not the case for the funds portfolios. Whatever the shortcomings of these techniques, they can give an approximate value of future profit and then an estimation of the carried interest.

Sahalman (1990) and Phalippou (2008) explain that the Black & Scholes formula can be used to calculate the fair value of a carry of 100% rate. In that case, they consider that the strike price is the initial invested capital, the value of the underlying asset is the fund value at the liquidation date, the maturity is the duration of the fund and then the volatility will be the volatility of investments' fund. Then they simply multiply the calculated call value by 20% to find the value of the carried interest. Knoll (2008) gives another proposal. He explained that the carry is equivalent to a call option; this is the right to receive 20% of the fund at maturity in exchange for paying the strike price equal to 20% of the initial value of the fund. The explanations are different but produce the same calculation.

Jääskeläinen, Maula and Murray (2007) examine the benefits' distribution and different fee structures used by hybrid funds. They evaluate the ability of government decision-maker to use these structures in order to improve expected yields and to attract private sector investors and professional managers to participate in the fund. They use a simulation of an investment process to model the performance of a fund whose portfolio consists of 15 investments. The development of each business scenario is modeled with a three level scenario tree. At each node of the tree, the venture capitalist evaluates the expected return of the investment.

According to Jääskeläinen, Maula and Murray (2007), when the company reaches the exit phase after a maximum of three rounds of financing, the investment is liquidated and the residual value is returned directly in cash to fund LPs. Once the hurdle rate is achieved by the fund's investment, the GPs can benefit from their share of profit defined by the carried interest (20% of profits).

Inspired by the work of Murray and Marriott (1998), Jääskeläinen et al. (2007) assume that the fund's portfolio of venture capital investments consists of 15 identical at an early stage investment. The development of a new business is simplified, it takes place in three stages: an initial funding stage, followed by two rounds of investment. At the end of each investment period, the company has four possible expressed values. In total, for any scenario, they get 64 (that is to say 4^3) different results at the end of the third stage. The final value of the company is determined by the success of each investment round as the portfolio companies evolve from one stage to another in a period of 6 years. To obtain the final values, they use multiples provided by Murray and Marriott (1998) from an international survey (Europe / USA) on venture capital funds at an early stage of the industry.

Metrick and Yasuda (2010) and Choi et al. (2011) propose an elaborated method to evaluate fund manager fees. They suppose that the "carried interest" can be likened to a basket call option on the portfolio of a limited partnership. They use detailed data on 249 funds raised between 1992 and 2006 in the United States. For every $ 100 of committed capital, they estimate that the average present value of compensation taken by a VC fund is $ 23.13 dollars (15.16 dollars for BO funds), which are divided into 14, 80 $ management fees (10.08 dollars for BO funds) and 8.33 dollars carried interest (5.08 dollars for BO funds). According to Metrick and Yasuda (2010), compensation funds on average capture 25% of the committed capital, thus lowering significantly the net return of investors capital.

Metrick and Yasuda (2010) note, however, that the basic structure is surprisingly invariant among the funds. It obeys massively to the model "2-20-8%." Changes in the application of this "rule" by the funds, however, can result in significant differences in commission.

2. The optional features of GPs compensation

2.1 Management fees

GPs receive "management fees" which are the only fixed part of their compensation. This part is easily calculated if we know the rate applied and the period during which it is granted. According to Metrick and Yasuda (2010), there are four methods to grant the fixed part of the GPs compensation. Historically, the most common method is to assess these costs as a fixed part of the committed capital (total contributed capital = invested capital + total fund fees). For example, if the management fees are estimated to an annual 2% of the committed capital, then the total costs throughout the fund’s life, supposedly 10 years, are estimated at 20% of the committed capital (2% * 10 years). If the fund does not support other expenses, the remaining 80% represent the invested capital.

In recent years, several funds adopted a digressive scale management fee with a lower rate of these costs after the investment period (the first 5 years). For example, a fund may set the management fee of 2% during the first 5 years of the investment period; this rate is reduced by 50 basis points for the last remaining years.

The third method of calculating management fees is to use a fixed rate but a different basis for each period. Indeed, this basis can be the committed capital for the first 5 years, against the invested capital for the next period.
Finally, the fourth method is to adopt both a declining rate and a different basis depending on the time. After the investment period, they use a lower rate for management fees and the invested capital rather than the committed capital as a calculation basis.

2.2 Carried interest

The variable performance-based compensation requires more interest because it represents the most important part of GP income in investment funds. This section focus on the specific features of the carried interest that demonstrate the optional characteristics and which justify the use of option pricing model to estimate its fair value. (Exhibit 1 presents a summary)

2.2.1 The carried interest rate

It represents the rate applied by fund to define the carried interest. The majority of funds; including Venture Capital (VC) and Buyout (BO) funds, uses a 20% level for the carry. In Metrick and Yasuda (2010) study, among the 98 VC funds, there is only one fund with a 17.5% carry level, three with a 25% rate and one fund with 30% rate, while the remaining funds of the sample used a 20% rate. In a more recent study by Johan and Najar (2011), the carried interest rates are between 17% and 25%.

In the literature, this rate is always supposed to be equal to 20%. The exact origin of this 20% rate is unknown, but some authors refer to the Venetian merchants of the Middle Ages, travelers speculative at the age of exploration and even the book of Genesis as a source.

2.2.2 The carried interest basis

The carried interest basis is freely determined. For firms, it may be the company's annual earnings. This basis can be limited only to net capital gains, with the exception of any other product. It is usually corrected by provisions that had to be made, as well as off balance sheet commitments that were given, as for example, the liability guarantees or bonds. Very often, the basis is not corrected to take into account the amounts actually received by the company.

For funds and according to L. 214-36, 8 CMF, the basis consists either of the fund net assets or fund products (capital gains, dividends, interest ...). There is not a fixed rule for funds to establish the carried interest basis. This variable, and all the carry terms are detailed in the fund rules.

Metrick and Yasuda (2010) consider that there are two different methods to calculate the carried interest basis. The first alternative, used by 93% VC funds and 84% BO funds of their sample, is to consider the committed capital as a carry basis. Otherwise, the rest of the sample funds use the invested capital.

Note that during the lifetime fund, a portion of the contributed capital is employed for the different fees as the management fees, the transaction fees, the monitoring fees…known as lifetime fees; the rest is invested in the fund's portfolio companies known as the invested capital. Thus, the committed capital is defined as the invested capital plus all the lifetime fees already paid by the fund.

This difference in defining the carried interest basis could have a significant effect on the value of the granted carried interest. Indeed, in the case of a fund making enough profit to distribute carries, the difference between the two methods could be measured by a difference in the value of carried interest equal to the carry level multiplied by the total cost supported by the funds during his lifetime.

2.2.3 The carried interest timing

As was stated above, the carried interest can be considered as an option on the fund investments portfolio, but investments in this type of portfolio have an unknown exit date. Metrick (2007) shows that on average the first wave of investment in Venture Capital has a holding period of five years, with a probability of exercise close to 20%. Metrick and Yasuda (2010) use these estimates for investments in VC funds, as well as those of BO funds. They believe that the exit date follows an exponential distribution with an exit rate $q = 20\%$ per year. They also assume that the output is not correlated with the performance of the underlying asset.

2.2.4 The investments volatility

To estimate the volatility of the fund's investments in VC funds, Metrick and Yasuda (2010) and Choi et al. (2011) refer to Cochrane (2005) work. In his study, Cochrane (2005) begins with the Capital Asset Pricing Model (CAPM) of expected VC investment incomes. There is always a problem to collect data of VC funds incomes. Indeed, these incomes are only observed in liquidation or financing event. To resolve this problem, Cochrane (2005) simultaneously estimated thresholds for IPO and bankruptcy liquidation. The establishment of these thresholds allows them to estimate the CAPM parameters that will involve averages and standard deviations of funds returns. For his sample, Cochrane (2005) estimated a volatility of 89% for VC investments which brings Metrick and Yasuda (2010) to choose a volatility of 90%.

For BO funds, there is a lack of data necessary for replicating Cochrane work. Metrick and Yasuda (2010), assume that BO funds often invest in listed companies (to make them out of the stock market) or in unlisted companies that are comparable to small business listed. Woodward (2004) found that the average beta of all BO funds is approximately equal to 1. This value is due to the fact that BO funds typically invest in companies with low Beta and try to improve it. Metrick and Yasuda (2010) estimate that investment volatility of BO funds will be the same as the public stocks of similar size with a Beta equal to 1. So to estimate the volatility of BO funds, they simply observe the volatility of listed companies that have the same size and have a unit beta. They therefore end up applying a volatility of 60% for BO funds according to the study of Campbell et al. (2001).

Generally, volatility depends on the maturity. Indeed, in the short term the volatility will be more important than in the long term for economic reasons influencing the market. According to option pricing models, the higher the volatility, the higher is the value of the option because high volatility reflects that at maturity, the portfolio value is more likely to exceeds its current value. On the other hand, in the case of an option, returns can lead to significant profits but loss, whether large or small, always lead to an option that will not be exercised.

The influence of volatility on the option value has led some studies to select a large range of volatility as was the case in Sahlman (1990) work. He tried in his study to estimate the present value of the carried interest as a percentage of the original capital for different values of volatility between 10% and 90%. Consequently, we propose in this paper to examine the sensitivity of the carried interest fair value to the volatility variations.

2.2.5 No arbitrage assumption

Arbitrage is the purchase of securities on one market for immediate resale on another in order to profit from a price discrepancy. In an efficient market, arbitrage opportunities cannot last for long. As arbitrageurs buy securities in the market with the lower price, the forces of supply and demand cause the price to rise in that market. Similarly, when the arbitrageurs sell the securities in the market with the higher price, the forces of supply and demand cause the price to fall in that market. The combination of the profit motive and nearly instantaneous trading ensures that prices in the two markets will converge quickly if arbitrage opportunities exist. Using the assumption of no arbitrage, financial economists have shown that the price of a derivative security can be found as the expected value of its discounted payouts when the expected value is taken with respect to a transformation of the original probability measure called the *equivalent martingale* measure or the *risk-neutral* measure. We can see Duffie (1996), Hull (1997), and Wilmott (1998) for more about risk-neutral pricing.

In the literature, evaluating carried interest model uses a risk-neutral approach, based on the no arbitrage condition. In reality, funds' investments are illiquid and the market is far to be perfect. Any time, this assumption is used in all models of stock option pricing, and conceptually does not require more than a discounted cash flow analysis of this type of action. It is important to note that the assessment is only applicable to an investor that can diversify unsystematic risk. The GP cannot do that. Indeed, he is unable to diversify the portfolio risk of his company. Thus, option-based valuation of carried interest should be interpreted as being proportional to the expected value to an outside "large" investor who owns a small claim on GPs revenue and should not be interpreted as an expected compensation to the GPs.

2.2.6 The possibility of hedging

A derivative (for example, a future contract, an option, a forward contract OTC, or swap) is a security whose value depends on the value of the underlying assets (eg, the value of a stock option is based in the valuation of the underlying stock to which it refers). Financial institutions and individual investors, dealing with options or other derivative securities are primarily concerned with hedging the risk of unfavorable market fluctuations, which may affect their potential performance and / or their position relative to the underlying asset. Specifically, many financial institutions and individuals use the listed option and / or OTC option to implement hedging strategies for the following reasons:

- Liquidity: to generate additional revenue from an existing stock position and create a measure of downside protection.
- Risk reduction: to reduce or eliminate exposure to the depreciation of the underlying shares and protect the value of their participation.
- Diversification: To reduce the risk of concentration by investing in a diversified and balanced portfolio.

If we assimilate the carried interest to an option, we can speak of a theoretical possibility of hedging these assets even if it is not a common practice. Indeed, the closest financial asset to the carried interest is the ESO (Employee Stock Option), since this is an option granted as an incentive tool. In addition, it is indexed to the company's shares. As carried interest, the ESO is inalienable, which means it cannot be sold, transferred or assigned to a third party, even when it has already been acquired (after the vesting period). Hedging by a contract with a third party is theoretically possible but there is little evidence that the coverage is widespread and systematic. According to Schizer (2000), the lack of coverage in practice may be due to reasons of reputation, transaction costs or "potentially crippling tax consequences" to the beneficiaries of such options.

Managers of a company may be forced to hold a certain number of shares of the company, whether such shares were allocated for free, or they result from exercise of stock options. In practice, it would be very difficult for the ESOs holder to cover his position by selling or shorting the underlying stock. Indeed, this approach is at odds with the main purpose of distributing ESOs (Alignment of executives and investors interests). On the other hand, it would be very difficult to find a counterpart in the market willing to buy such option. Nevertheless, in theory, it is possible to implement a hedging mechanism for any financial instrument among others, the carried interest.

3. The non-marketability criteria

As Employee Stock Option, the Carried interest can be considered as a non marketable and non transferable option for a period of time. This criterion is a very important factor in estimating the fair value of financial assets [Bouvet et al. (2009), Mun (2004), Brenner et al. (2001), Hull and White (2004), Huddart and Lang (1996)]. In addition, all previous works to estimate the carried interest have ignored this criterion that may have a significant impact on the estimated fair value.

First of all, an asset is considered illiquid if it takes more than three days to sell it. Some consider that the non-marketability is an extreme form of illiquidity and others think that the non- marketability and illiquidity are two concepts completely different, since illiquidity is a "fact" because it reflects how easy to exchange a financial asset in the market, and the non- marketability is "right" because it reflects a trade ban. Moreover, there is some confusion in the designations used in some papers where the term "illiquidity" is used to refer to a temporary impossibility of tradability on the market.

In legal circles, the discount due to the non-transferability poses two main types of controversies: (i) the first is whether there is a legitimacy to apply this discount with a particular financial asset, and (ii) the second relating to the appropriate level of this discount. For example, to evaluation a non-marketable asset, Emory (1995) examines the United States case law regarding to the applicability of marketability discounts to determine fair value in case of dissident litigation. He finds that there is considerable disagreement as to whether a marketability discount should be applied or not.

Regarding to the discount value, some studies estimate that the discount is calculated by comparing the price of an asset during a period in which it is negotiable with the price of that asset when it is no longer negotiable. Other studies compare the prices of two simultaneously possible options on the same underlying asset, where one is negotiable on the market and the other is not. The third group of studies compares the transaction prices of assets that are readily marketable asset prices with otherwise similar with limited marketability. [Hertzel and Smith (1993), Wruck (1989), Johnson (1999), Maher (1976), Silber (1991)…]

Moreover, these methods mainly based on the comparison have been widely criticized because they concern only a minority of financial assets that meet their criteria. Another more sophisticated technique proposed by Longstaff (1995) is to treat the discount as a loss of a put option. He considered the restricted share holder as a buyer of a look-back Put option. He sets an upper bound on the illiquidity value, recognized as the non-marketability. He assumes that an investor has the ability to determine the most opportune time to invest and when he must sell his shares to achieve maximum gain. Moreover, if the investor is restricted from selling the shares for a given period, in which case he waives his right to sell his shares at the most appropriate time and thus achieving his maximum gain. The difference between discounted price from the sale of shares after the restriction period and discounted price from sale at their maximum value in this case represents the value of the non-marketability. Using conventional techniques in option pricing literature and value of certain actions parameters, Longstaff (1995) estimates the discount for lack of liquidity based on the restricted period and the standard deviation of stock returns. He concludes that the marketability discount can be economically relevant, even when the period of illiquidity is relatively short.

He reported that the discount was between 25% and 35% given typical liquidity restrictions on private placements. Plainly, volatility and time explain widely the variation discount.

Several studies were inspired by the approach of Longstaff (1995), trying each time to establish the most effective model for calculating the cost of non-marketability.
From previous studies on evaluating the non- marketability discount, we see that in the case of carried interest, it would be more appropriate to apply the option method.
Thus, the fair value of the carry is equal to the estimated value of a call option to which we subtract the cost of illiquidity.

Exhibit 1

Optional features of the carried interest

	Ordinary call option	Carried Interest	The impact on the carried interest value
The Strick price	the strike price fixed in the option contract	Any point of comparison as: hurdle rate (minimum rate of return), or the occurrence of an event that could trigger the delivery of the carried interest.	The higher the performance of the benchmark or target, the lower the probability of carried interest allocation. (Srike price ↗ CI ↘)
The underlying asset	Quoted price of the underlying asset	The value of an index or a financial performance measure or operations. The carried interest is indexed to earnings, cash flow earnings or even market share.	The greater the underlying index performance (to which the carried interest is attached), the greater will be the value of the carried interest. (Underlying asset price ↗ CI ↗)
Interim payments	Dividends	All the intermediate flows related to carried interest	The greater the interim payments, the lower will be the value of carried interest after the payment. (Dividends ↗ CI ↘)
Duration	At the issue date, the contracts are for three, six or nine months	Generally the fund's lifetime is estimated between 10 and 12 years.	When the remaining life of the carried interest is long, its value will be higher. The lifetime is generally the second most important factor of the options pricing model. (lifetime ↗ CI↗)
Uncertainty	The volatility of underlying assets returns	The uncertainty of the underlying index performance to which the carried interest is attached. In other words, it is the volatility of investment funds returns.	The greater is the uncertainty (or volatility), the greater is the carried interest value. Volatility is generally the most important factor in the options pricing method. (Volatility ↗ CI ↗)

4. Monte Carlo simulations

Monte Carlo simulation for option pricing was introduced by Boyle (1976). This approach is to generate trajectories of stochastic processes in order to estimate possible changes of the option during its lifetime. The option value is then expressed as an expectation of different scenarios on a given date. The simulation is very useful when calculating option values for which there is no analytical formula and in the presence of several state variables. This technique is very suitable for continuous processes or process with jumps.

Monte Carlo simulation has the distinction of being simple to use in the valuation of derivatives. This method is to generate many possible trajectories of the underlying asset and calculate the final values of the derivative for each path and then take their average and up to date. Thus, the Monte Carlo simulation is to generate a trajectory of the underlying asset in the world risk-neutral, then calculated from this trajectory the value of the derivative, repeat these steps a few times, and finally calculates the average of the derivative and update.

The Monte Carlo simulations are fairly simple and easy to implement and change, they are quite flexible and can be applied to a variety of cases where it is difficult to solve partial differential equations, or when the tree method is not appropriate. The simulations produce the desired degree of precision and are effective to price options on a large number of underlying assets.

Determining the present value of the carried interest is analogous to pricing a basket call option. Although a basket option can be priced approximately in a closed form. The use of Monte Carlo simulation method allows us to compute the present value of the carried interest. It is the only method that takes in account the particularity of investment fund portfolio:

- the number of assets in the portfolio changes over time
- the exit date of each investment is unknown

In the simulation, we assumed that:

- The fund makes a predetermined number of investments with equal size. The number of investments is set to match the value of the fund sample in Metrick andYasuda (2010).
- The fund makes the investments at the beginning of the year during the first 5 years.
- The investment duration for the portfolio company follows an exponential distribution with the instantaneous hazard rate of 20% since the average holding period of venture capital investments is about 5 years.

- Any investments not yet exited are forced to be liquidated at the end of the 12 year (maximum lifetime of each investment)
- Under these assumptions, we make 10.000 Monte Carlo simulations and obtain the average of the carried interest.
- We calculate the Put value, which represent the non-marketability discount.
- The fair value of the carried interest is the value of the Call option obtained by the simulation reduced by the value of the Put option.
- Varying the parameters value of the Call and Put options (volatility, duration, hurdle rate…) and analyze the impact on the fair value of the carried interest.

4.1 The dynamic of the investments portfolio value

To determine the value of carried interest, it was assumed that it is an option based on the fund's portfolio. In practice, this type of option is called basket option. Basket options are options whose underlying is a portfolio of assets. This option does not take into account the weighted sum of assets taken independently but relies instead on a portfolio composed of assets with equal or unequal weights. The correlation between assets in the basket options used to affect the volatility and therefore the price obtained by summing individual options on each asset.

$$P = \widehat{E}\left(S_i S_j\right) = F_i F_j e^{\rho_{ij}\sigma_i\sigma_j T} \qquad (1)$$

Where Ê denotes the expected value in a risk-neutral world

P: Value of basket at time T

n: The number of assets

S_i: The value of the ith asset at timeT

F_i: The forward price of the ith asset for a contract maturing at time T

$\sigma_{i:}$ The volatility of the ith asset between time zero and time T

ρ_{ij}: Correlation between returns from the ith and jth asset

A simple approximation method assumes that the basket spot itself is a log-normal process with drift μ and volatility σ driven by a Wiener Process. The dynamics is represented by:

$$\frac{dS_i}{S_i} = \mu_i d_t + \sigma_i dW_i \quad \text{pour i = 1,…, d} \qquad (2)$$

Geometric Brownian motion is a continuous stochastic process whose logarithm follows a Brownian motion. It is applied in the mathematical modeling of certain courses in financial markets.

It is mainly used for option pricing because a quantity that follows a geometric Brownian motion takes any positive value and only changes the basic random variable is significant. The geometric Brownian motion is a reasonable approximation of changes in share price.

This type of continuous process that cannot be built by machines working by nature of discrete values, there must be a prior discretization into N intervals of length Δt = T / N. Be used for this is the Euler approximation of a broadcast system to rewrite the above equation as follows:

$$S(t+\Delta t) - S(t) = r\, S(t)\, \Delta t + \sigma S(t) \varepsilon \sqrt{\Delta t} \quad (3)$$

where ε is a random number from a normal distribution standard. However, it is more accurate to simulate ln (S) rather than S. the application of Itô lemma to calculate the dynamics of ln (S), which is written:

$$d \ln(S) = (r - \frac{\sigma^2}{2}) dt + \sigma dz \quad (4)$$

And which can be discretized using the following approximation:

$$\ln(S(t+\Delta t)) - \ln((S(t)) = (r - \frac{\sigma^2}{2}) \Delta t + \sigma \varepsilon \sqrt{\Delta t} \quad (5)$$

Or, equivalently, by passing to the exponential of each member of the above equation

$$S(t+\Delta t) = S(t) e^{(r - \frac{\sigma^2}{2}) \Delta t + \sigma \varepsilon \sqrt{\Delta t}} \quad (6)$$

4.2 The dynamic of the investments exit date

According to Choi et al. (2011), the fund life follows an exponential distribution with the instantaneous hazard rate of λ = 20% (because the average holding period is 5 years then 1/5 = 20%).

Corresponds to an exponential model:

X is a random variable defining the life of a phenomenon. If the life expectancy of the phenomenon is E (X), then X has the probability density:

- $f(t) = 0$ if $t < 0$ (7)
- $f(t) = \frac{1}{E(X)} e^{-\frac{t}{E(X)}}$ if $t \geq 0$.

We say that X has an exponential distribution with parameter $\lambda = \frac{1}{E(X)}$

In practice, we always measure an exponential waiting time, lifetime, etc.

In fact the random variable X is expressed in units of time (second, hour, day, year, etc.) and then λ is a frequency. We say that a random variable X follows an exponential distribution with parameter λ> 0 if we have the formula:

$$P\,[X > t] = e^{-\lambda t} \quad (8)$$

For investment fund, we suppose that for each investment i, the duration d_i follows an exponential distribution with the instantaneous hazard rate of λ as follow:

$$f(d_i) = \lambda e^{-\lambda d_i} \quad (d_i \geq 0) \quad (9)$$

In practice, LPs and GPs don't have any control on the exit timing of each investment. The exit opportunities arrive more or less exogenously. On the other hand, we suppose that d_i is independent of the investment performance which is certainly false, but it is rather difficult to consider this kind of correlation in modeling the value of an investment portfolio.

In general fund have a defined "start date" and typically have a fixed life of ten years that can be extended by a pre-set number of defined periods (e.g., two one-year periods) upon agreement of the investors. So the investment duration is controlled. It cannot exceed twelve years.

4.3 The simulation process stages

To enhance the carried interest received by GPs in investment funds, we use the Monte Carlo simulations. These simulations are performed using software designed to integrate into an Excel spreadsheet. The most famous software simulations are "Crystal Ball" and "@ risk".

As part of this research, we chose to run simulations on the software "Crystal Ball". This product is an excellent tool for creating and executing stochastic simulation models.
we will describe in what follows, the different steps of the simulation process.
Exhibit 2 represents a simple flowchart for the simulation model. The following description helps identify the tasks involved in each step of the simulation process.

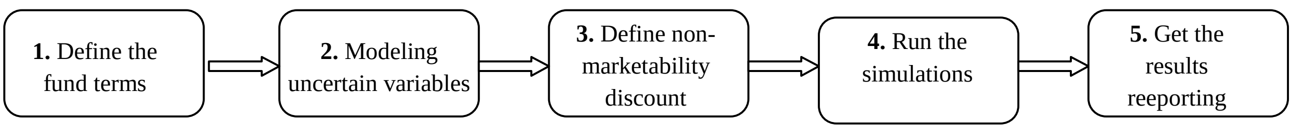

Exhibit 2: Simulation process steps

Step 1: define the fund terms

The first step consists to define the fund terms for each set of trial. We define the initial value of different fund variables: the total invested capital and the allocation for each investment, the number of investment, the carried interest rate, the hurdle rate, the management fees rate, the correlation and the volatility of portfolio investment and the average life of each investing and investment volatility.

We generate fourteen different scenarios for venture capital fund and similarly for Buyout funds. The initial scenario is the benchmark portfolio: each parameter takes a value inspired by literature and previous work. In other scenarios, we vary each time one of these parameters to achieve a sensitivity analysis of the carried interest value.

It is assumed that the funds invest in projects throughout the investment period which is typically the first five years of the lifetime fund. All the investments are spread over this period. We suppose that the investments frequency for VC and BO funds during the investment period follows the example presented in Exhibit (3). We suppose that the VC benchmark fund invested 100 monetary units in a portfolio of 25 identical investments. The investments have a volatility of 90% with an exit rate of λ=20%. The BO benchmark fund spends 105 monetary units for 15 investments with a volatility of 60% and the same exit rate. We also require an assumption about the correlation of any pair of investments. For VC fund, we don't have any indication in the literature about the correlation between portfolio investments so we adopt the estimate of Metrick and Yasuda (2010) who set it at 50%. For BO funds, we follow Campbell et al. (2001) who define a pair-wise correlation of 20%. Concerning the profit-sharing rules, it is assumed that both types of funds (VC or BO) grant a carried interest of 20% and LPs require a preferred return of 8% (hurdle rate).

Exhibit 3: Frequency of investments funds

Investment period	1	2	3	4	5	Total investments number
Investments number - VC	8	7	6	3	1	25
Investments number - BO	4	4	3	2	2	15

Step 2: Modeling uncertain variables

In this step, we simply select the distribution of random variables of our model. For this simulation, the two "assumption" variables, reported to the Crystal Ball software are the duration and the future value of each investment.

According to the literature, it was agreed that the duration of each investment follows an exponential law. For benchmark portfolio, we set an exit rate $\lambda = 20\%$ (average investment lifetime = 5 years).

The second random variable of this simulation is the future value of each investment that follows a geometric Brownian motion. On the other hand, it is assumed that the investments of each fund are generally correlated as funds choose to manage projects that belong to almost the same industry in order to take advantage of the experts on whom they rely in all investments. The Crystal Ball software offers the possibility to define the correlation between the random variables of the model and allows us to draw the correlation matrix (table 3).

[Insert Table 3 About Here]

For the benchmark portfolio, we assume that the correlation between investments is 50% for venture capital fund and 20% for Buyout funds. We realize later simulations with different values of correlation and we discuss the implications of using different estimates for the pair-wise correlation.

Step 3: determine non-marketability discount

This step defines the non-marketability discount input variables. Indeed, if we assume that the carried interest is a share of future fund profits, the carried interest is considered as a call option on the future value of the portfolio fund. The underlying investments are not listed on the market. In addition, the beneficiary of carried interest (GPs) has no ability to liquidate his right, to transfer it, or to exchange it in the market. So the fair value of the carried interest will be underpriced.
According to previous literature, we decided to calculate the non-marketability discount as the value of a put option with the same characteristics of the carried interest.

Step 4: Running simulations

It is proposed to achieve 100 000 iterations for each investment. The software gives each time a value for the investment lifetime according to the exponential distribution. This value is controlled as it should not exceed 12 years (maximum investment duration). Then the software calculates the final value of each investment at liquidation date and the total net asset value of the fund portfolio is the total present value of all investments. For each investment, the profit is calculated and updated.

It is proposed to calculate the carried interest in two different ways. The first method assumes that the carried interest is awarded by investment (case "deal by deal") and the second method consists to calculate de carry under the "whole fund".

In practice, the fund partners must decide whether carried interest will be distributed on a deal by deal or on a whole fund basis. Under the deal by deal model, returns are generally calculated for each investment, and the manager receives its carried interest as profits are realized on the particular investment. In contrast, under a whole fund model, the manager does not receive carried interest distributions until the investors receive distributions equal to their total capital contributions to the entire fund and a preferred return on all such contributions. Assuming that a fund incorporates a so called ''claw-back''[1] feature, both the deal by deal model and the whole fund model should result in the same aggregate sharing of profits over the life of the fund, with the only variable being the timing of receipt of such profits by the manager—earlier for a deal by deal model and later for a whole fund model. Of course, in this paper, we suppose that there is no claw-back clause in order to illustrate why a claw-back is required to preserve the proper aggregate carried interest percentage in a deal by deal model.

In absence of a claw-back clause, a manager is not required to give back any portion of such current income carried interest following subsequent investment losses. As a result, this waterfall is also quite pro-manager, particularly when employed by a fund generating significant current income.

Finally, we calculate the "underpriced" present value of carried interest (PV CI) which represents an estimate of the fair value of carried interest in case of "whole fund" reduced by the non-marketability discount.

[1] ''claw-back'' clause : At the liquidation of the fund, if the manager has received carried interest and either (a) the investors have not received their specified preferred return on their total contributions to the fund through that point in time or (b) the total carried interest paid to the manager to that point in time exceeds 20 % of the aggregate profits of the fund, the manager will pay to the investors the greater of (i) the amount of carried interest the manager has received in excess of 20 % of the aggregate profits of the fund or (ii) the amount required to provide the investors their preferred return, but usually, with respect to amounts provided in both (i) and (ii), never in excess of the aggregate amount of carried interest the manager has actually received, net of taxes the manager has paid on such carried interest.

Step 5: The recovery results

Gets the Crystal Ball reporting that contain forecasts statistics. For each scenario, we collect the average of carried interest for "deal by deal fund" and "whole fund", in addition, the “underpriced” fair value of carried interest (PV CI). Table (2) summarizes the data for venture capital funds and buyout funds.

5. Simulations outputs

5.1 management-fees outputs

Table (1) summarizes the different simulation results of management-fees granted to GPs as a fixed compensation. As noted earlier, there are different methods applied by investment funds to calculate management-fees. The traditional method consists on applying a fixed annual rate on the committed capital for the duration of fees payment, which generally corresponds to the lifetime funds. To analyze the sensitivity of management-fees to the applied rate, the calculations were performed for a 2% rate (the most common) and then with increased and decreased rate (Panel A-case 1). In the general case, the total fair value of management-fees collected by GPs after 10 years of managing a fund with a committed capital of 100 *currency units* will be 16.22-*cu*. The increase of 5% of the applied rate increases the present value of management fees of 4.05-*cu*, a drop of 5% reduces the management fees by 4.06-cu.

The second method of calculating the management fees is the decreasing fee schedule (Panel A-case2). It involves applying an initial annual rate during the investment period (the first 5 years) and applying a lower rate for subsequent years. For this case, we performed the calculations in three different scenarios. Initially, we assume a management fees rate of 2% that decreased to 1.5% after 5 years. The fair value of management fees in this case decreases by 11% compared to the general model. Moreover, the use of 2.5% rate and 2% rate respectively for the investment period and the liquidation period leads to an increase of the fair value by nearly 14%.

The third alternative of fees schedule uses a constant rate, but changes the basis for this rate from committed capital for the first five year to the invested capital for the last five year (Panel B-case1). The application of this model for a 2% management fees rate led to a decline of 9% of the total fair value perceived by GPs.

The last type of fees schedule uses both a decreasing percentage and a change from committed capital to net invested capital after the investment period (Panel B-case 2). We suppose a 2% rate for the first five year and 1.5% rate for the last five year. This method induces a decrease of 18% in the fair value of management fees compared to the general case.

[Insert Table 1 About Here]

5.2 Carried interest outputs

Table 2 summarizes the simulation results of the carried interest fair value. In this table, there are three types of carried interest: the carried interest paid to each investment for "deal by deal" fund (CI "dbd"), the carried interest calculated on the total portfolio for "whole fund" (CI "WF") and finally underpriced carried interest (PV CI) which is the fair value of the carried interest after deduction of the non-marketability discount.

In the strict deal by deal model, each deal stands alone, and the profits and losses of each deal are insulated from the profits and losses of other investments made by the fund. Under this model, the manager receives carried interest from proceeds of an individual investment as soon as each investor recoups its capital contribution and corresponding preferred return attributable to such investment. The manager is entitled to keep any carried interest distributions regardless of whether the fund's other investments are (or even the fund as a whole is) profitable. This model essentially provides a manager a series of independent options on investment profit. Managers only have the possibility of being rewarded for making good investments and have no possibility of being punished for making bad ones. In the absence of a clawback clause, we note that the carried interest of deal by deal funds is much higher than that calculated on the total portfolio fund (whole fund). In the case of a venture capital-type "deal by deal" fund, the carried interest is on average 24% higher than that accorded by a "whole fund". The difference is more pronounced in the case of buyout funds, as it can reach up to 44%. This difference is due to the fact that the carried interest granted by deal by deal fund does not take into-account the case of unprofitable investments. Nevertheless, in the basic whole fund model distribution waterfall, each investor must recoup its total capital contributions to the fund and receive a specified preferred return on those total contributions before the manager is entitled to receive any carried interest. So, the application of the "deal by deal" model requires the need for a clawback clause to ensure that GPs receives only their share of profit, calculated on the total portfolio.

In this simulation work, we first analyze the benchmark cases. We assume two types of funds: venture capital fund and a buyout fund. The parameter values of the benchmark venture capital (buyout) model are: 20% exit probability, 20% carry level, 100 *cu* (105 *cu*) of invested capital, 90% (60%) total volatility, 50% (20%) pairwise correlation, 8% hurdle rate and 25 (15) investments.

In this general case of venture capital fund, the fair value of carried interest is 8.34 *cu* which represents 8.34% of invested capital and 7.25% of total committed capital by investors taking into account all costs supported by the fund. On the other hand, the fair value of the carried interest in buyout fund is 5.58 *cu*. Moreover, these values do not reflect the illiquid nature of carried interest. Indeed, if one assumes that the carried interest can be approximated to a basket option, this option is nontransferable and unchangeable in the market. So the real fair value of carried interest must consider the non marketability discount calculated us a put option. From simulation results, the underpriced fair value of carried interest is 6.25*cu* (3.74 *cu*) which only represent 6.25 % (3.56%) of invested capital in venture capital fund (buyout fund).

[Insert Table 2 About Here]

We also examine the effects on the different present values of carried interest of perturbing seven model parameter values: carry rate, carry basis, hurdle rate, exit probability, number of investments, investments volatility and pairwise correlations between portfolio investments.

The value of carried interest is considerably sensitive to the variation of the applied carry rate (Figure 1). Reducing the carry rate by 10% leads to a lower carried interest by 0.55 *cu* in the case of a venture capital Whole fund. Otherwise, an increase to 25% rate increases the present value of nearly 3 *cu* and a 10% increase in the rate passed away at 4.8 *cu*. We observe the same reaction for buyout fund compensation: a 10% rate decline reduces the fair value of the carried interest of 1.47 *cu*, while a 10% increase causes an increase of 1.9 *cu*. The fair value of carried interest, net of non marketability discount follows the same trend in the case of venture capital fund or funds buyout funds.

[Insert Figure 1 About Here]

In Figure 2, the fair value of carried interest increases when investments volatility increases. This result is confirmed for venture capital and buyout funds. A 10% decrease of the volatility investment led to a decline in fair value of net carried interest of 0.42 *cu* for venture capital fund and 0.65 *cu* for buyout fund.

[Insert Figure 2 About Here]

Similarly, the correlation between investment has a positive effect on the fair value of carried interest but the magnitude of the variation is lower (Figure 4). Indeed, an increase of 10% of the correlation between investments causes an appreciation of the carried interest of 0.34 cu (0.24 *cu*) for venture capital fund (buyout fund).

[Insert Figure 4 About Here]

Regarding the exit probability (Figure 5), it was assumed that a λ = 20% in the general case corresponds to an average life of 5 years for each portfolio investment. For the sensitivity analysis, we proceed to a decrease and an increase of this probability. Thus, for a λ of 10% (an average lifetime of 10 years) the value of carried interest increases significantly. For example, for venture capital fund type "whole fund", the fair value changes from 8.34 *cu* to 11.42 *cu* and the net fair value changes from 6.25 *cu* to 8.05 *cu*. On the other hand, an exit probability of 30% (an average lifetime of 3.33 years) leads to a decrease of the net carried interest fair value by 6% (17%) for venture capital funds.

[Insert Figure 5 About Here]

Assuming smaller portfolios, we find that the number of investments managed by the fund has a negative effect on the carried interest value (figure 6). For venture capital fund, reducing the portfolio investments from 25 to 15 leads a small increase (0.08 cu) of the carried interest fair value. For buyout fund, a portfolio of 10 investments instead of 15 investments generates a higher carried interest by of 8%. With smaller number of investments, the overall fund portfolio is less well diversified, so the volatility of the portfolio is higher and the option value (carried interest) is higher.

[Insert Figure 6 About Here]

The value of carried interest is also affected by its basis (figure 7). Indeed, using an invested capital basis, means that the GP does not receive his carried interest until the investors recover all their invested capital (in addition to the preferred return) but committed capital basis implies that we must consider all the contributed capital by investors, including expenses paid in management fees, transaction fees and monitoring costs. It is assumed that these costs represent nearly 12% of invested capital according Metrick and Yasuda study (2010). Simulations show that the carried interest decreases by nearly 15% when the calculation is based on committed capital. This result is also confirmed for net carried interest. The effect is more pronounced for buyout funds since the decrease of the carried interest fair value is about 23%.

[Insert Figure 7 About Here]

Finally, we examined the reaction of the carried interest value with respect to the preferred return rate (hurdle rate) required by investors (figure 3). We found that when the hurdle rate increases, the fair value of carried interest decreases. An increase of 1% hurdle rate caused a slight decrease of 1.8% of the fair value of the carried interest. Conversely, decreasing the minimum rate of return from 8% to 7% increases the carried interest earned by GPs of 0.35 *cu* and 0.28 *cu* of the net fair value. Similarly, the hurdle rate has a negative effect on the carried interest distributed by buyout funds. It is estimated that a higher hurdle rate (9%) causes a decrease of 6.68% of the net fair value; however a drop of the hurdle rate to 7% induces a nearly 5% increase in net carry value.

[Insert Figure 3 About Here]

Conclusion

In conclusion, under the deal by deal model, returns are generally calculated for each investment, and the manager receives its carried interest as profits are realized on the particular investment. In contrast, under a whole fund model, the manager does not receive carried interest distributions until the investors receive distributions equal to their total capital contributions to the entire fund and a preferred return on all such contributions. Assuming that a fund must incorporates a ‘‘claw-back’’ clause, because both the deal by deal model and the whole fund model should result in the same aggregate sharing of profits over the life of the fund.

According to these simulations we found that fund managers earn a total of nearly 22.47% of the invested capital in venture capital funds and approximately 20% for buyout funds throughout the lifetime fund.

On the other hand, this paper include a sensitivity study helps to highlight the effect of different characteristics of the fund, the investments portfolio and profit sharing rules, on the present value of carried interest granted to fund managers. We note that the carried interest reacts like a call option relative to the volatility, correlation and the duration of the underlying asset. In addition, the simulations confirm that when the terms of contracts governing the carried interest distribution are favorable for GPs, the fair value of the expected carry increases. Finally, when investors are less demanding, the preferred return is lower and the GPs saw their share of profit increasing.

The last idea of this paper concerns under pricing of carried interest fair value. The non-marketability discount is estimated to 24% for venture capital fund and near 31% for buyout fund.

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Table 1
Estimating Management fees

Mgt fees rate	2%
Committed capital	100
Duration	10
Discount rate	5%

Mgt fees rate	↓0.5%
Committed Capital	100
Duration	10
Discount rate	5%

-Panel A. Changing Management fees rate

There are four different methods to grant the management fees. The general model (Panel A-case 1) is the most common consist to apply a fixed rate on the committed capital. The second method (Panel A-case 2) is the digressive model. In this case, the management fees rate decreases after the investment period (fist 5 years). We test these two methods for different rates.

Case 1-General Model of Mgt fees

Year	fee level	Mgt fees	PV Mgt fees
1	2%	2	2.00
2	2%	2	1.90
3	2%	2	1.81
4	2%	2	1.73
5	2%	2	1.65
6	2%	2	1.57
7	2%	2	1.49
8	2%	2	1.42
9	2%	2	1.35
10	2%	2	1.29
total fees			**16.22**

Year	fee level	Mgt fees	PV Mgt fees
1	1.5%	1.5	1.50
2	1.5%	1.5	1.43
3	1.5%	1.5	1.36
4	1.5%	1.5	1.30
5	1.5%	1.5	1.23
6	1.5%	1.5	1.18
7	1.5%	1.5	1.12
8	1.5%	1.5	1.07
9	1.5%	1.5	1.02
10	1.5%	1.5	0.97
total fees			**12.16**

Year	fee level	Mgt fees	PV Mgt fees
1	2.5%	2.5	2.50
2	2.5%	2.5	2.38
3	2.5%	2.5	2.27
4	2.5%	2.5	2.16
5	2.5%	2.5	2.06
6	2.5%	2.5	1.96
7	2.5%	2.5	1.87
8	2.5%	2.5	1.78
9	2.5%	2.5	1.69
10	2.5%	2.5	1.61
total fees			**20.27**

Case 2-Digressive Model of Mgt fees

Year	fee level	Mgt fees	PV Mgt fees
1	2%	2	2.00
2	2%	2	1.90
3	2%	2	1.81
4	2%	2	1.73
5	2%	2	1.65
6	1.50%	1.5	1.18
7	1.50%	1.5	1.12
8	1.50%	1.5	1.07
9	1.50%	1.5	1.02
10	1.50%	1.5	0.97
total fees			**14.43**

Year	fee level	Mgt fees	PV Mgt fees
1	2.5%	2.5	2.50
2	2.5%	2.5	2.38
3	2.5%	2.5	2.27
4	2.5%	2.5	2.16
5	2.5%	2.5	2.06
6	2%	2	1.57
7	2%	2	1.49
8	2%	2	1.42
9	2%	2	1.35
10	2%	2	1.29
total fees			**18.49**

Year	fee level	Mgt fees	PV Mgt fees
1	3%	3	3.00
2	3%	3	2.86
3	3%	3	2.72
4	3%	3	2.59
5	3%	3	2.47
6	2.5%	2.5	1.96
7	2.5%	2.5	1.87
8	2.5%	2.5	1.78
9	2.5%	2.5	1.69
10	2.5%	2.5	1.61
total fees			**22.54**

fees level	2%
Committed Capital	100
Duration	10
Discount rate	5%

fees level	2%
Invested Capital	80
Duration	10
Discount rate	5%

-Panel B. Changing Management fees basis

The third method of granting management fees consist on changing the management fees basis for the committed capital to the investment capital after the investment period (Panel B-case 1). Finally the fourth method (Panel B-case 2) involves reducing the management fees rate and changing the basis after the investment period. We suppose that the investment capital represent 80% of the committed capital.

Case 1-Changing basis model

Année	fee level	Mgt fees	PV Mgt fees
1	2%	2	2,00
2	2%	2	1,90
3	2%	2	1,81
4	2%	2	1,73
5	2%	2	1,65
6	2%	1,6	1,25
7	2%	1,6	1,19
8	2%	1,6	1,14
9	2%	1,6	1,08
10	2%	1,6	1,03
total fees			**14,79**

Case 2- Digressive & changing basis model

Année	fee level	Mgt fees	PV Mgt fees
1	2%	2	2,00
2	2%	2	1,90
3	2%	2	1,81
4	2%	2	1,73
5	2%	2	1,65
6	1.5%	1,2	0,94
7	1.5%	1,2	0,90
8	1.5%	1,2	0,85
9	1.5%	1,2	0,81
10	1.5%	1,2	0,77
total fees			**13,37**

Table 2

Results from Crystal Ball simulations

This table summarizes outputs of Monte Carlo simulations on Crystal Ball software. We realized fourteen scenarios of each type of funds: venture capital fund and buyout fund. CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount. We test the carried interest fair value evolution after the variation of each optional parameter: Carried interest rate, investments volatility, the hurdle rate, the correlation between portfolio investments, the investment exit date, the number of portfolio investments and the carried interest basis.

Venture capital	Parameters	CI « dbd »	CI « WF »	PV CI	Buyout	Parameters	CI « dbd »	CI « WF »	PV CI
CI rate	20%	10,42	8,34	6,25	CI rate	20%	7,86	5,58	3,74
	10%	9,64	7,79	5,83		10%	5,91	4,11	2,83
	25%	13,05	11,35	8,49		25%	9,95	6,2	4,14
	30%	14,27	13,14	9,87		30%	11,59	7,49	4,53
Volatility	90%	10,42	8,34	6,25	volatility	60%	7,86	5,58	3,74
	80%	10,02	7,75	5,83		50%	6,37	4,46	3,09
	70%	8,51	7,05	5,32		70%	8,24	6,47	4,16
Hurdle rate	8%	10,42	8,34	6,25	Hurdle rate	8%	7,86	5,58	3,74
	7%	10,75	8,69	6,53		7%	8,12	5,83	3,92
	9%	10,96	8,19	6,13		9%	7,64	5,23	3,49
correlation	50%	10,42	8,34	6,25	correlation	20%	7,86	5,58	3,74
	30%	9,27	7,16	5,39		10%	7,77	5,17	3,42
	60%	10,98	8,85	6,59		30%	8,04	5,94	3,98
Exit date	5 years	10,42	8,34	6,25	Exit date	5 years	7,86	5,58	3,74
	3.33 years	10,02	7,67	5,86		3.33 years	5,21	4,1	2,73
	10 years	12,82	11,42	8,05		10 years	12,47	7,87	5,42
N° of invest	25	10,42	8,34	6,25	N° of invest	15	7,86	5,58	3,74
	15	10,45	8,42	6,34		10	8,01	5,81	4,06
CI basis	Invested capital	10,42	8,34	6,25	CI basis	Invested capital	7,86	5,58	3,74
	Committed capital	10,17	7,11	5,29		Committed capital	6,82	4,24	2,88

Figure 1

Sensitivity to the carried interest rate

This figure represents the evolution of the carried interest value for different level of carry rate (10%, 20%, 25% and 30%). CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.

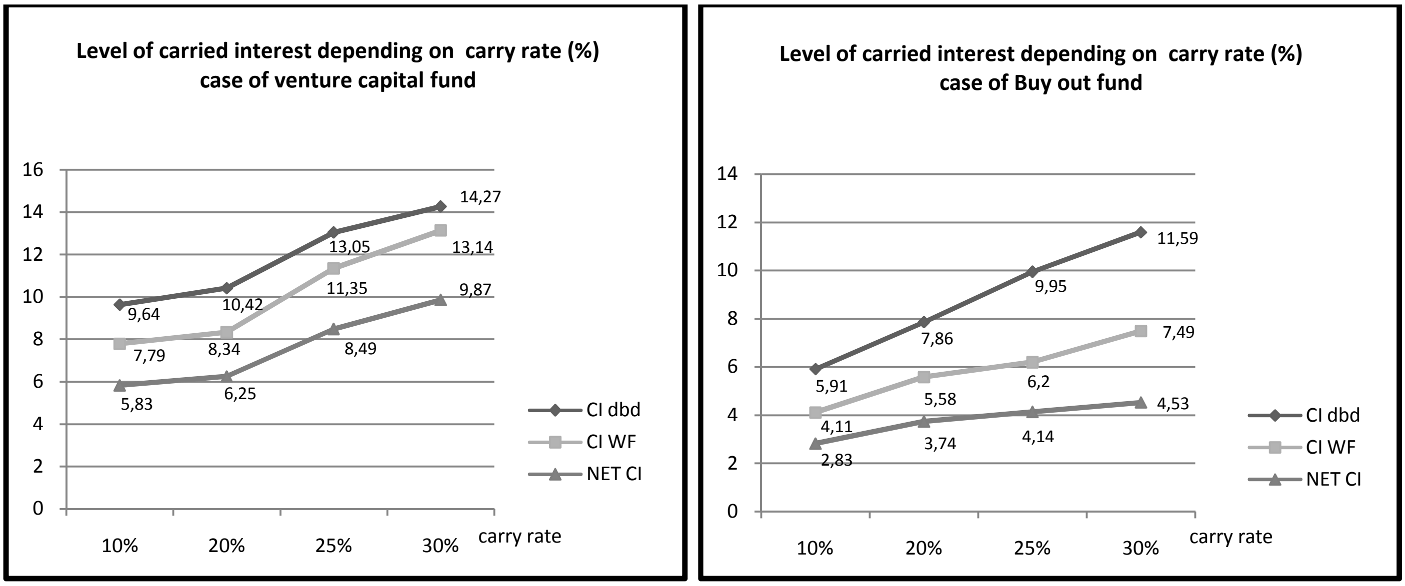


Figure 2

Sensitivity to the volatility of investments

The present value of carried interest is represented for different value of investment volatility. The annual volatility of individual investments is set to: 70%, 80% and 90% for venture capital funds and to 50%, 60% and 70% for buyout funds. CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.

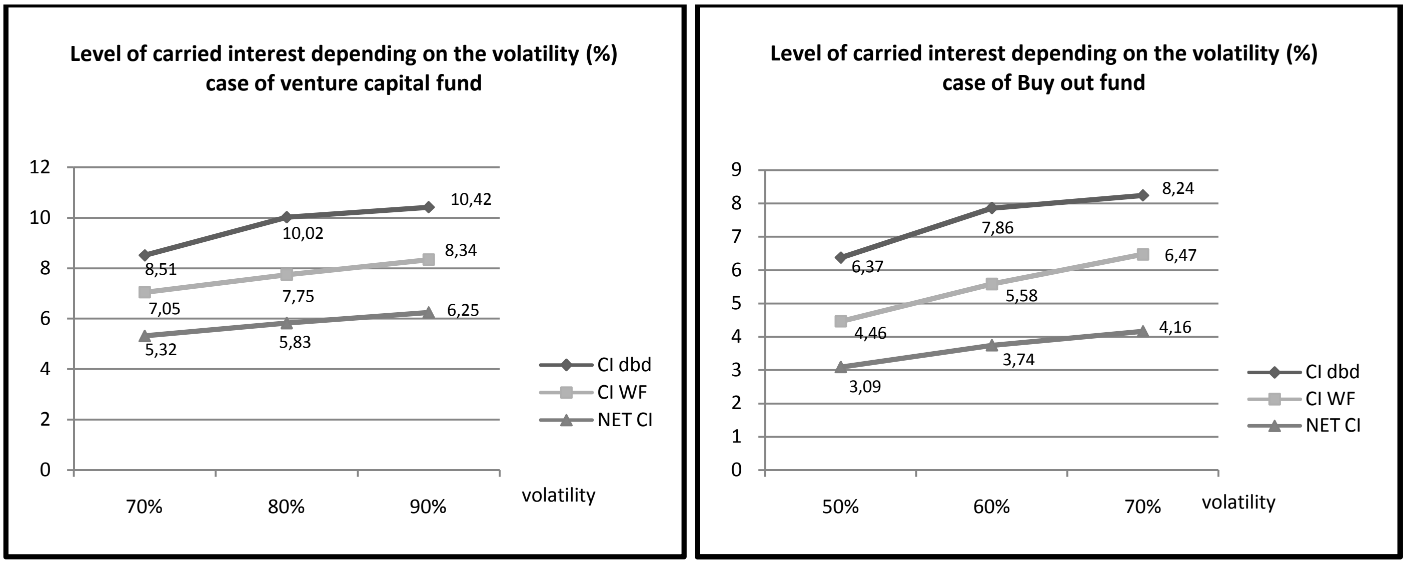
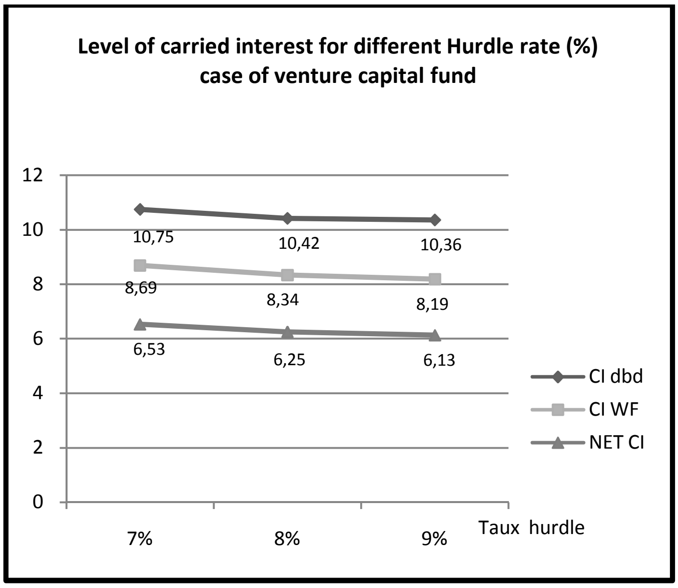


Figure 3
Sensitivity to the hurdle rate

We simulate the fair value of the carried interest for different level of the hurdle rate required by LPs (7%, 8% and 9%). CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.



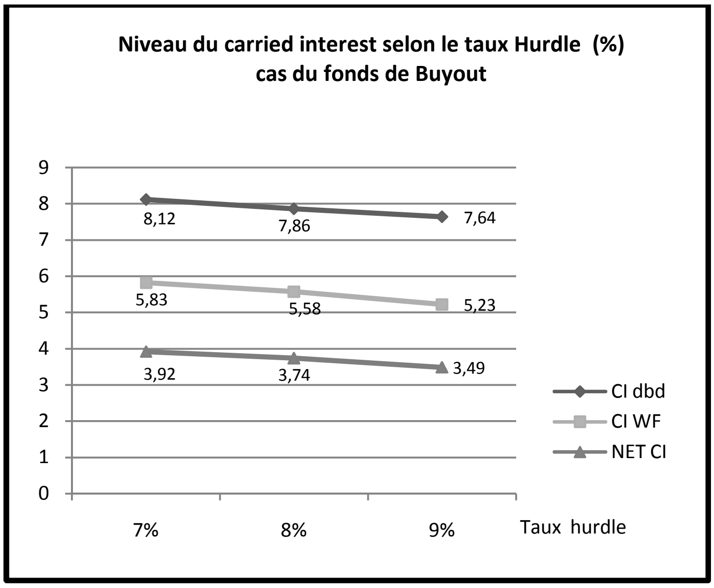
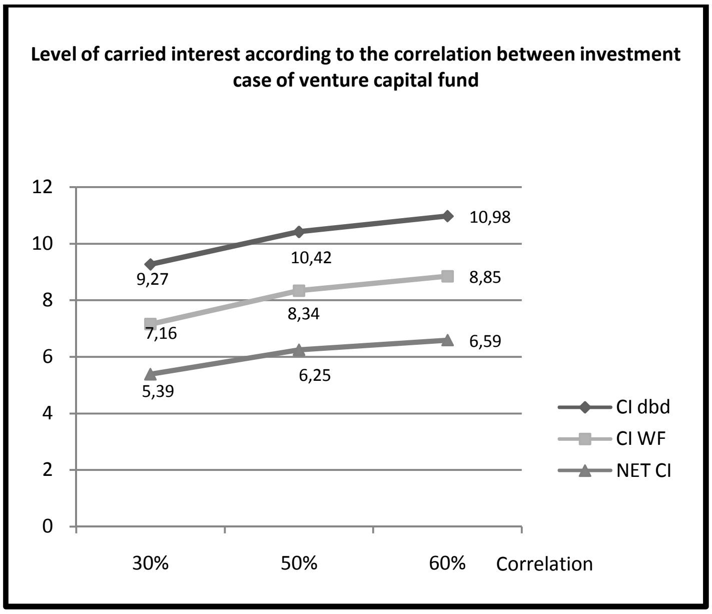


Figure 4
Sensitivity to the correlation between investments

We simulate the fair value of carried interest for different value of investments correlation. For the venture capital fund, the correlation values are 30%, 50% and 60%. For the buyout fund, the correlation values are 10%, 20% and 30%. CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.



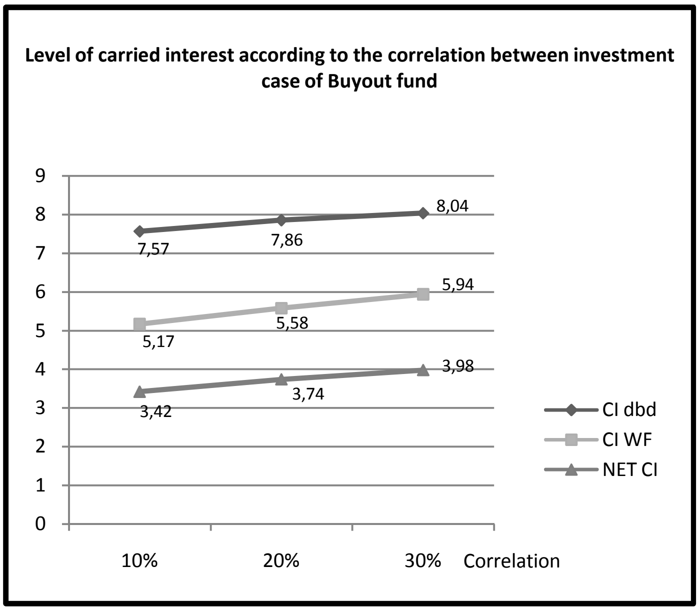


Figure 5
Sensitivity to the exit dates

The present value of the carried interest is represented for different value of investment average holding period. The simulations concern three different value of the holding period [3,3 years (λ=30%), 5 years (λ=20%) and 10 years (λ=10%)]. CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.

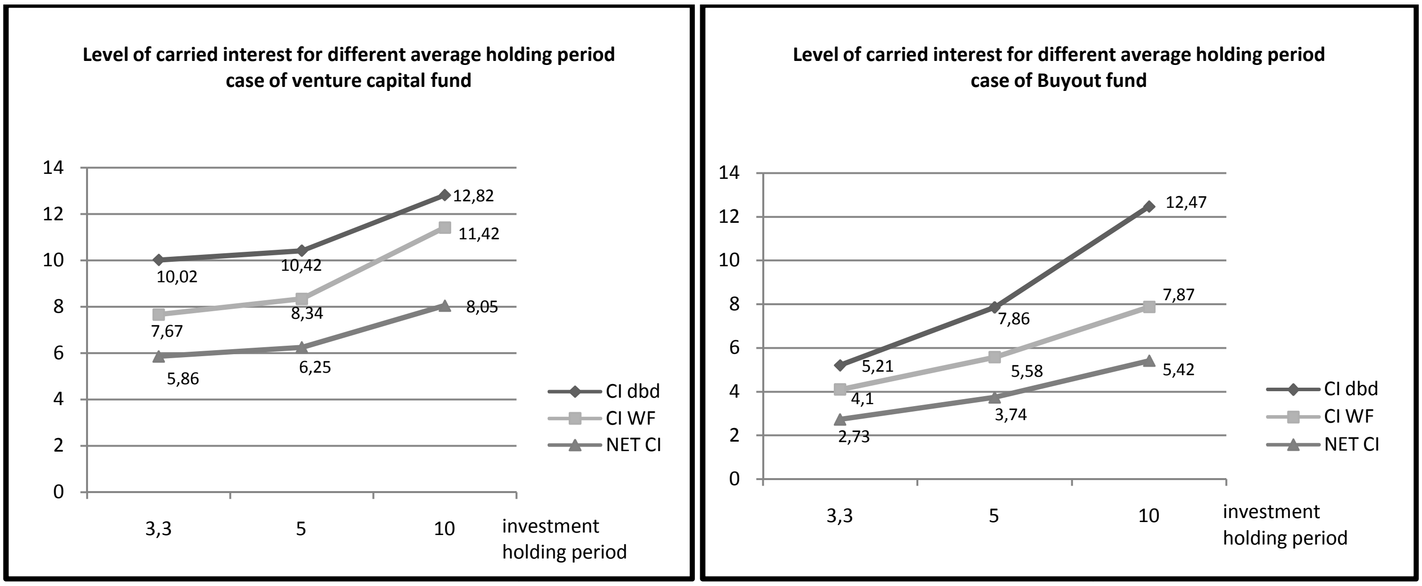


Figure 6
Sensitivity to the number of fund's portfolio investments

Determining the effect of the investments portfolio number on the fair value of the carried interest consists on simulating two different scenarios for venture capital fund :25 investments and 15 investments, and two scenarios for Buyout fund: 15 investments and 10 investments . CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.

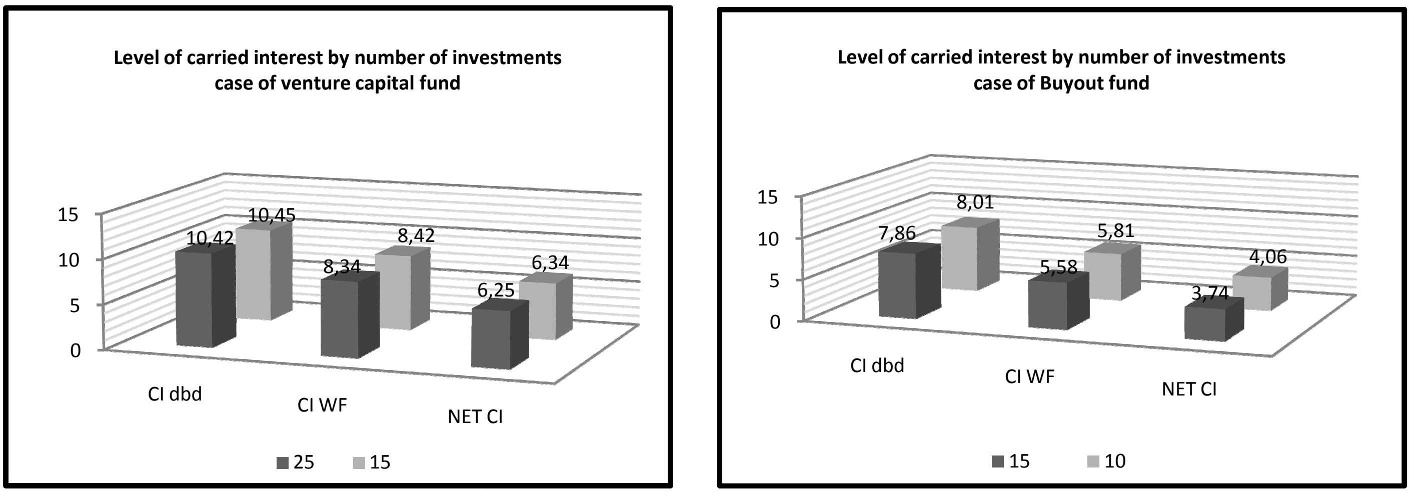
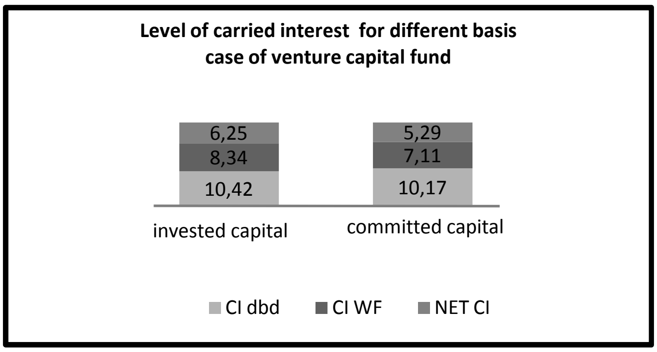


Figure 7
Sensitivity to the basis of carried interest

We simulate the carried interest value for different basis. The two different methods to calculate the carried interest basis is the invested capital and the committed capital. The invested capital represents the sum of funds invested in the portfolio. The committed capital includes invested capital and all the fund fees (management fees, monitoring fees, transaction fees…). CI dbd is the present value of the carried interest calculated in a deal by deal fund. CI WF is the fair value of the carried interest calculated in a whole fund. NET CI represents the fair value of the carried interest after reducing the non-marketability discount.



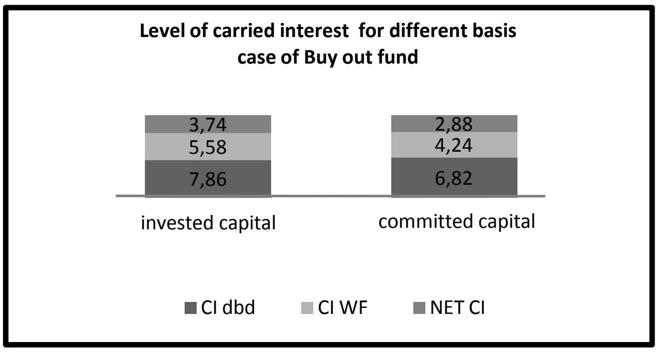


Table 3 (Panel A)
Correlation Matrix of Venture capital investments fund

Load the matrix

	B24 (Feuil1)	C24 (Feuil1)	D24 (Feuil1)	E24 (Feuil1)	F24 (Feuil1)	G24 (Feuil1)	H24 (Feuil1)	I24 (Feuil1)	J24 (Feuil1)	K24 (Feuil1)	L24 (Feuil1)	M24 (Feuil1)	N24 (Feuil1)	O24 (Feuil1)	P24 (Feuil1)	Q24 (Feuil1)	R24 (Feuil1)	S24 (Feuil1)	T24 (Feuil1)	U24 (Feuil1)	V24 (Feuil1)	W24 (Feuil1)	X24 (Feuil1)	Y24 (Feuil1)	Z24 (Feuil1)
B24 (Feuil1)	1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
C24 (Feuil1)		1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
D24 (Feuil1)			1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
E24 (Feuil1)				1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
F24 (Feuil1)					1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
G24 (Feuil1)						1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
H24 (Feuil1)							1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
I24 (Feuil1)								1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
J24 (Feuil1)									1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
K24 (Feuil1)										1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
L24 (Feuil1)											1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
M24 (Feuil1)												1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
N24 (Feuil1)													1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
O24 (Feuil1)														1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
P24 (Feuil1)															1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
Q24 (Feuil1)																1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
R24 (Feuil1)																	1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500	0.500
S24 (Feuil1)																		1.000	0.500	0.500	0.500	0.500	0.500	0.500	0.500
T24 (Feuil1)																			1.000	0.500	0.500	0.500	0.500	0.500	0.500
U24 (Feuil1)																				1.000	0.500	0.500	0.500	0.500	0.500
V24 (Feuil1)																					1.000	0.500	0.500	0.500	0.500
W24 (Feuil1)																						1.000	0.500	0.500	0.500
X24 (Feuil1)																							1.000	0.500	0.500
Y24 (Feuil1)																								1.000	0.500
Z24 (Feuil1)																									1.000

Table 3 (Panel B)
Correlation Matrix of Buyout investments fund

Load the matrix

	B24 (Feuil1)	C24 (Feuil1)	D24 (Feuil1)	E24 (Feuil1)	F24 (Feuil1)	G24 (Feuil1)	H24 (Feuil1)	I24 (Feuil1)	J24 (Feuil1)	K24 (Feuil1)	L24 (Feuil1)	M24 (Feuil1)	N24 (Feuil1)	O24 (Feuil1)	P24 (Feuil1)
B24 (Feuil1)	1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
C24 (Feuil1)		1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
D24 (Feuil1)			1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
E24 (Feuil1)				1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
F24 (Feuil1)					1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
G24 (Feuil1)						1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
H24 (Feuil1)							1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200	0.200
I24 (Feuil1)								1.000	0.200	0.200	0.200	0.200	0.200	0.200	0.200
J24 (Feuil1)									1.000	0.200	0.200	0.200	0.200	0.200	0.200
K24 (Feuil1)										1.000	0.200	0.200	0.200	0.200	0.200
L24 (Feuil1)											1.000	0.200	0.200	0.200	0.200
M24 (Feuil1)												1.000	0.200	0.200	0.200
N24 (Feuil1)													1.000	0.200	0.200
O24 (Feuil1)														1.000	0.200
P24 (Feuil1)															1.000